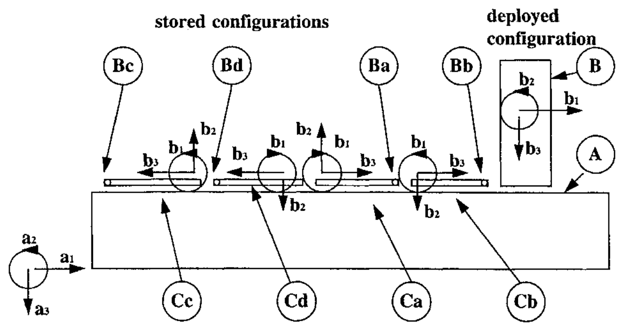

Fig. 1 Stored and deployed configurations.

Table 1 Admissible solutions

Configuration	$e_1\sqrt{3}$	$e_2\sqrt{3}$	$e_3\sqrt{3}$	θ
Ca	1	−1	1	$2/3\pi$
Cb	−1	−1	−1	$2/3\pi$
Cc	1	1	−1	$2/3\pi$
Cd	−1	1	1	$2/3\pi$

configurations of B in A, namely, the ones called *Ca*, *Cb*, *Cc*, and *Cd* in Fig. 1. In connection with each of these, one may pose the question of Ref. 2, namely, "Is there an axis . . . ?" In accordance with Euler's theorem, the answer is positive when applied to each of *Ca*, *Cb*, *Cc*, and *Cd*, and involves a point P fixed in both A and B, a line L passing through P and fixed in both A and B, a unit vector $\boldsymbol{\lambda}$ parallel to L, and an angle θ indicating the measure of rotation of B in A about L from the stored configuration to the deployed configuration. Equations (1–4) in Ref. 1 (and, similarly, the results of Ref. 3, originally used to conduct this analysis) enable the calculation of θ and e_i, defined as $e_i = \boldsymbol{\lambda}\cdot\boldsymbol{a}_i$ $(i = 1, 2, 3)$. For example, Eq. (1) leads, in connection with the *Ca* configuration (and, in fact, in connection with each of the indicated configurations), to $\theta = \pm(2/3\pi + n\pi)$ $(n = 0, 1, \ldots)$, and the value of θ appearing in Table 1 is chosen from an engineering standpoint. Moreover, in connection with the *Ca* configuration, R_{ij} $(i, j = 1, 2, 3)$, the direction cosines relating the deployed configuration of B to the *Ca* (stored) configuration are $R_{11} = 0$, $R_{12} = -1$, $R_{13} = 0$, $R_{21} = 0$, $R_{22} = 0$, $R_{23} = -1$, and $R_{31} = 1$, $R_{32} = 0$, $R_{33} = 0$, obtained with the aid of Fig. 1, by inspection; and when substituted in Eqs. (2–4) in Ref. 1, lead to the values reported in the first row of Table 1. Lastly, points *Ba*, *Bb*, *Bc*, and *Bd* shown in Fig. 1 play the role of P in connection with configurations *Ca*, *Cb*, *Cc*, and *Cd*, respectively.

Keeping in mind that the wing has to be aerodynamically deployed, and assuming that the forward velocity of the aircraft is essentially in the $-\boldsymbol{a}_1$ direction, as described in Ref. 2, one must rule out configurations *Cc* and *Cd* as inadequate for aerodynamical deployment; and is left with configurations *Ca* and *Cb*. In Ref. 2 these are called *LD* and *UD* configurations, respectively, and are the ones associated with Eqs. (1–5).

In conclusion, the choice of the *LD* and *UD* configurations in Ref. 2 as admissible is the outcome of an engineering elimination process, in which all possible results suggested by the theoretical analysis have been considered; and not because the wing was assumed to be rectangular.

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Comment on "Analytic V Speeds from Linearized Propeller Polar"

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IN a recent Technical Note, Lowry[1] presented several equations that had been previously given by Laitone.[2] For example, Lowry's Eq. (12) for the maximum level flight speed V_m can be mathematically clarified as follows when $T = D$:

$$T = E + FV^2 \qquad D = GV^2 + H/V^2$$
$$(G - F)V^4 - EV^2 + H = 0 \tag{1}$$
$$V_m^2 = \frac{1}{2(G - F)}\left[E + \sqrt{E^2 - 4(G - F)H}\right]$$

Since Lowry[1] defines $(G - F) > 0$, Eq. (1) is identical to Lowry's Eq. (12), and is identical to Laitone's[2] Eq. (11), if one notes that F must be negative for any fixed-pitch propeller since its thrust must decrease as the velocity increases.

As shown by Laitone,[2] the thrust for any fixed-pitch propeller can be approximated by the theoretical relation

$$T = \frac{P_p}{V_p}\left[\frac{3}{2}\phi - \frac{\sigma}{2}\left(\frac{V}{V_p}\right)^2\right] = \frac{P_{\text{av}}}{V} \tag{2}$$

where V_p is the velocity corresponding to peak or maximum power available $(P_{\text{av}})_{\max} = P_p$. Therefore, this ideal fixed-pitch propeller has

$$F = -(\sigma/2)(P_p/V_p^3) \quad \text{and} \quad T = 0 = P_{\text{av}}$$
$$\text{when} \quad V_0 = \sqrt{3\phi/\sigma}V_p \tag{3}$$

The altitude correction factor $\phi \le \sigma = \rho/\rho_0$, used by Lowry,[1] is discussed in Refs. 3 and 4, which show the effect of various approximations to ϕ, and compare the performance predictions from Eq. (2) with various methods.

Similarly, for $F = -\sigma P_p/2V_p^3$ and $\cos\alpha = 1$, Lowry's[1] Eq. (14) for the speed for fastest rate of climb is identical to Laitone's[2] Eq. (13), and his Eq. (16) for the speed for maximum angle of climb is identical to Laitone's Eq. (20). Assuming $\cos\alpha \approx 1$ is justified for any conventional airplane with a fixed-pitch propeller since the thrust vector remains nearly parallel to the velocity vector for $C_L < 1$. Lowry[1] also states, following his Eq. (18), that $F < 0$ would be "intuitively correct." However his Eqs. (5) and (18) do not give a realistic estimate of the increase in $|F|$ when the blade pitch angle β is decreased. Most analyses show that $F \sim 1/\beta^3$, and this is in agreement with Eq. (3) since a decrease of β directly decreases V_p. The dependence of V_p upon β can be established from Eq. (3) by

$$T = 0 \quad \text{when} \quad [\beta_0 - \tan^{-1}(V_0/r\omega)] \approx \alpha = 0$$

Therefore, when $\phi = \sigma = 1$, Eq. (2) is defined for any selected β_0 and engine power (bhp) by the following:

$$V_0 = r\omega\tan\beta_0, \qquad V_p = V_0/\sqrt{3}, \qquad P_p = \eta\,(\text{bhp}) \tag{4}$$

The accuracy of this approximation for V_p and P_p in Eq. (2)

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is of course dependent on the estimate of η (the propulsive efficiency), which is usually 0.85 from V_m to $V_* = (H/G)^{(1/4)}$ (the velocity for the airplane's maximum lift–drag ratio). It is shown in Ref. 2 that $V_p = V_{mp} = V_*/3^{1/4}$ for $(DV)_{min}$ is not a realistic choice, even though Eqs. (1) and (2) predict V_{mp} produces the maximum rate of climb, because C_{Lmp} becomes too large for the airplane drag polar's validity, as shown in Fig. 1 of Ref. 2. Also, Fig. 2 of Ref. 2 shows that $V_p = (V_m)_{max}$ would be a suitable fixed-pitch propeller design criteria since it produces V_{max} and a good rate of climb, as shown in Fig. 1 of Ref. 4. As shown in Ref. 2, any decrease in β and V_p would increase the rate of climb, but decrease the maximum speed. Another consideration is that the decrease in β and V_p increases the takeoff thrust, since $E = (3\phi P_p)/(2V_p)$ also increases (see Fig. 3 in Ref. 2).

However, if neither the rate of climb, nor the takeoff distance, are the prime consideration, then $V_p = V_{max}$ produces the best compromise for a fixed-pitch propeller. As indicated in Fig. 2 of Ref. 2, $V_p > V_{max}$ produces an inferior propeller with lower climb rates. The optimum $V_p = V_{max}$ is given for $\sigma = 1$, by

$$DV = GV_p^3 + H/V_p = P_p = \eta\ (\text{bhp})$$

The resulting quartic equation is easily evaluated by the following first iteration [see Eq. (11), Ref. 3]:

$$V_{max} = V_p \approx (P_p/G)^{1/3}[1 - (H/G)(P_p/G)^{-4/3}]^{1/3} \qquad (5)$$

The resulting values for V_p and P_p allow Eq. (2) to be used to estimate the performance for $V < V_p$ and $\sigma < 1$ for the optimum fixed-pitch propeller.

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Reply by the Author to E. V. Laitone

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IN his Comments on Lowry's[1] Note, Laitone[2] cites three instances in which Lowry's equations are "identical to" equations given by Laitone[3] in earlier work. We disagree, on grounds that two equations cannot be identical when they contain variables standing for essentially different physical constructs. Lowry's entire development was based, as the title of his Note indicates, on the linearized propeller polar. The propeller polar concept, linearized or not, is never mentioned in Laitone's[3] article or in either of his follow-up articles[4,5] cited in Ref. 2.

When the subject Note was submitted for publication we were not aware of Laitone's work. If Laitone's[3–5] articles had been brought to our attention they would have been cited in our Note as interesting work along similar lines, just as was done in the case of similar work by von Mises.[6] Each of these three investigators assumed identical quadratic forms for the variation of thrust T with airspeed V:

$$T(V) = E + FV^2 \qquad (1)$$

Since there is no argument over the form of the drag polar

$$D(V) = GV^2 + (H/V^2) \qquad (2)$$

(though there may be disagreement over the range of speeds for which it accurately holds), the form of Eq. (1) is the precise and sole reason behind the identical forms of V-speed solutions given by the three investigators. Identity of form under varying interpretations of coefficients E and F is not, in our opinion, tantamount to the equations themselves being identical. "At night all cows are gray," said Hegel; likewise, under open-ended substitution of variables, all equations read the same: $A = B$.

The physical meaning of any one of these investigator's interpretation of coefficient E or F does differ from the others. von Mises's interpretation employs representative blade elements. Laitone's interpretation uses static thrust and maximum level flight speed at sea level, assumed to occur at the point where power available TV also peaks. Lowry's interpretation uses the linearized propeller polar parameters, slope m and intercept b. The eclectic reader is free to decide which interpretation is most reasonable, compelling, and useful to his or her purposes.

Because propellers may behave strangely at very low forward speeds and small advance ratios J, we preferred not to use static thrust as a basic parameter in our theory. It is, for instance, not quite the case, as mentioned by Laitone,[2] that "thrust must decrease as the velocity increases." In the regime of ordinary flight his assertion is almost certainly true (and we have used it there), but at very low (or no) forward speed, propeller blade stall characteristics may result in positive values for dC_T/dJ, and hence, possibly to increases of thrust with increased airspeed. For examples, see thrust coefficient graphs in the classic article of Hartman and Biermann.[7]

Another point of difference between the treatments of Lowry[1] and Laitone[2] is exemplified in the latter's assertion "the estimate of η (the propulsive efficiency) which is usually 0.85 from [maximum level flight speed to speed for maximum lift/drag ratio]." Unless supplemented with rpm data, Lowry's scheme makes no use of the concept of propeller efficiency. As written elsewhere, Lowry's[8,9] method is a reduced description involving only one relation, the propeller polar, instead of the pair of relations afforded by the full story, graphs of thrust coefficient and power coefficient as functions of advance ratio.

A further point of difference resides in Laitone's[3] statement "For a fixed-pitch propeller, Oswald has indicated that for most aircraft the best performance is obtained when the maximum or peak-power is available at the maximum velocity for steady level flight at sea level . . ." Laitone's use of this idea shows not only a difference in technique (we have found use of relations that one hopes should hold, or of speed ratios, to be much less fertile than a constructive approximate theory of propeller action), but also of basic motive. The purpose of the linear propeller polar approach to flight performance prediction is to not presume the characteristics of some ideal aircraft, or of most aircraft, but simply to give airplane owners, fleet operators, and manufacturers a useful tool for evaluating performance of the actual aircraft they have at hand. The propeller polar bootstrap approach is currently being used for that practical purpose by some small aircraft manufacturers.

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